STAT 3115 – Homework 2 Solutions

Section 2.2

30. (a) $P_{3,8} = \frac{8!}{5!} = 336$.

(b) $\binom{30}{6} = 593,775$.

(c) $\binom{8}{2}\binom{10}{2}\binom{12}{2} = 83,160$.

(d) $P(\text{two of each variety}) = \dfrac{\binom{8}{2}\binom{10}{2}\binom{12}{2}}{\binom{30}{6}} = 0.14$.

(e) $P(\text{all one variety}) = \dfrac{\binom{8}{6} + \binom{10}{6} + \binom{12}{6}}{\binom{30}{6}} = 0.00196$.

38. (a) $P(\text{exactly two 75W}) = \dfrac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} = 0.2967$.

(b) $P(\text{all same type}) = \dfrac{\binom{4}{3} + \binom{5}{3} + \binom{6}{3}}{\binom{15}{3}} = 0.0747$.

(c) $P(\text{one of each type}) = \dfrac{\binom{4}{1}\binom{5}{1}\binom{6}{1}}{\binom{15}{3}} = 0.264$.

Boxcar Defects.

(a) $P(\text{both systems are defective}) = \dfrac{\binom{2}{2}\binom{3}{0}}{\binom{5}{2}} = 0.1$.

(b)

$$P(\text{at least one is defective}) = 1 - P(\text{none defective}) = 1 - \frac{\binom{2}{0}\binom{3}{2}}{\binom{5}{2}} = 0.7.$$

Poker Chips.

(a) $P(\text{one of each color}) = \dfrac{\binom{3}{1}\binom{5}{1}\binom{1}{1}}{\binom{9}{3}} = 0.18$

(b) $P(\text{no blue chips}) = \dfrac{\binom{5}{0}\binom{4}{3}}{\binom{9}{3}} = 0.0476$.

(c) $P(\text{at least one blue chip}) = 1 - P(\text{no blue chips}) = 1 - 0.047 = 0.953$

Section 2.4

45. (a)

$$\begin{aligned} P(A) &= 0.106 + 0.141 + 0.200 = 0.447 \\ P(C) &= 0.215 + 0.200 + 0.065 + 0.020 = 0.5 \\ P(A \cap C) &= 0.200 \end{aligned}$$

(b)

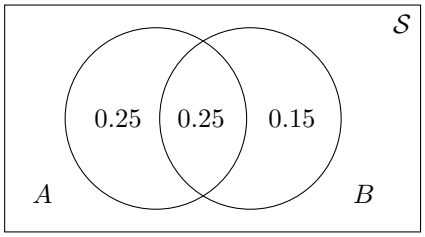
$$P(A \mid C) = \frac{P(A \cap C)}{P(A)} = \frac{0.2}{0.5} = 0.4$$
$$P(C \mid A) = \frac{P(A \cap C)}{P(C)} = \frac{0.2}{0.447} = 0.4474$$

$P(A \mid C)$ is the chace that someone has blood type A given that they are in ethnic group 3. $P(C \mid A)$ is the chance that someone is in ethnic group 3 given that they have blood type A.

(c) Let E_1 be the even that the individual belongs to Ethnic Group 1.

$$P(E_1 \mid B') = \frac{P(E_1 \cap B')}{P(B')} = \frac{0.082 + 0.106 + 0.004}{P(\text{type O}) + P(\text{type A}) + P(\text{type AB})} = 0.211$$

47. First, fill out a completed Venn diagram.



(a) Given that someone has a Visa, what's the chance they have a MasterCard?

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{0.25}{0.5} = 0.5$$

(b) Given that someone has a Visa, what's the chance that they *don't* have a MasterCard?

$$P(B' \mid A) = \frac{P(B' \cap A)}{P(A)} = \frac{0.25}{0.5} = 0.5$$

(c) Given that someone has a MasterCard, what's the chance that they have a Visa?

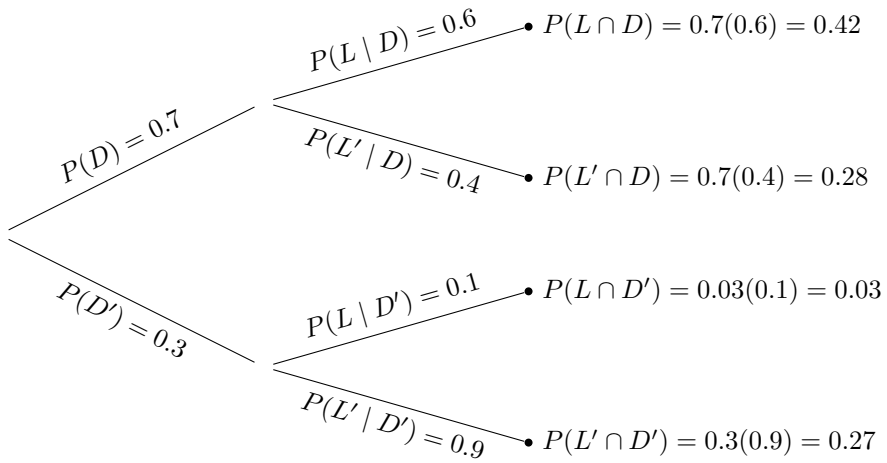
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.25}{0.40} = 0.625$$

(d) Given that someone has a MasterCard, what's the chance that they *don't* have a Visa?

$$P(A' \mid B) = \frac{P(A' \cap B)}{P(B)} = \frac{0.15}{0.40} = 0.375$$

(e) $P(A \mid \text{at least one card}) = \dfrac{P(A \cap (A \cup B))}{P(A \cup B)} = \dfrac{0.5}{0.65} = 0.769$

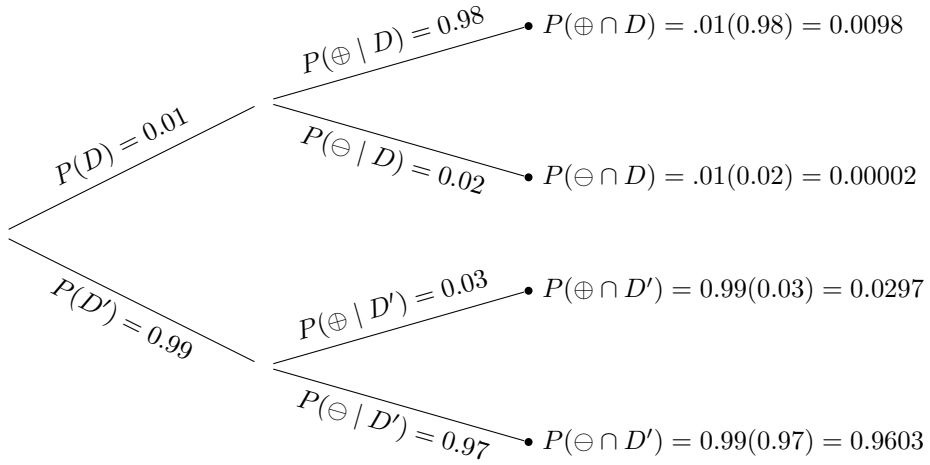
60. Let D be the event the plane is discovered and L the event it has an emergency locator. Draw a completed tree diagram to find necessary probabilities.



(a) $P(D' \mid L) = \dfrac{P(D' \cap L)}{P(L)} = \dfrac{P(L \mid D')P(D')}{P(L)} = \dfrac{0.03}{0.03 + .42} = 0.0\overline{6}$

(b) $P(D \mid L') = \dfrac{P(D \cap L')}{P(L')} = \dfrac{0.28}{0.28 + 0.27} = 0.51$

Olympic Drug Testing. Let D be the event that an athlete uses the drug and let $\oplus$ be the event that the athelete tests positive (and subsequently, $\ominus$ be the event that the athletes test negative). Use a tree diagram to find all necessary probabilities.



(a) $P(\oplus) = P(D \cap \oplus) + P(D' \cap \oplus) = 0.0395$

(b) $P(D \mid \oplus) = \dfrac{P(\oplus \cap D)}{P(\oplus)} = \dfrac{0.0098}{0.0098 + .0297} = 0.248$